Int. J. Heat Mass Transfer. Vol. 17, pp. 1437–1442. Pergamon Press 1974. Printed in Great Britain

RADIATION EFFECT ON HYDROMAGNETIC CONVECTION IN A VERTICAL CHANNEL

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(*Received* 31 *August* 1973)

Abstract—The effect of radiation on the combined free and forced convection flow of an electrically conducting fluid inside an open-ended vertical channel and permeated by a uniform transverse magnetic field is considered. Closed form solutions for the velocity, temperature and the induced magnetic field are obtained in the optically thin limit case when the wall temperatures are varying linearly with the vertical distance. It is found that radiation tends to increase the rate of heat transport to the fluid thereby reducing the effect of natural convection. Velocity and the induced magnetic field increase and the temperature difference between the wall and the fluid decreases with increase in the radiation parameter. In the unstable situation corresponding to heating of the channel from below, both radiation and magnetic field exert stabilizing influence on the flow.

NOMENCLATURE

B, induced magnetic field;
B_0, applied magnetic field;
C, constant defined by equation (8);
c_p, specific heat at constant pressure;
F, radiation parameter defined by equation (16);
g, gravitational acceleration;
L, half width of channel;
p, pressure;
q_R, radiative heat flux;
T, temperature;
T^*, temperature defined by equation (1);
T_{w0}, wall temperature at $z = 0$;
v, velocity.

Greek symbols

α, thermal diffusivity;
β, volumetric expansion coefficient;
θ^*, the temperature difference $T^* - T_{w0}$;
μ, magnetic permeability;
ν, kinematic viscosity;
ρ, reference density;
σ, electrical conductivity.

Subscript

w, value at the wall.

INTRODUCTION

SEVERAL investigations have been carried out on problems of heat transfer in electrically conducting liquids permeated by electromagnetic fields. Such studies are of importance in the design of MHD generators, cross-field accelerators, shock tubes and pumps. A comprehensive review of these problems is given by Romig in [1]. Siegel [2], Perlmutter and Siegel [3], and Alpher [4] presented detailed analysis of forced convection heat transfer to an electrically conducting liquid flowing in a channel with a transverse magnetic field. Convective flow in a vertical channel was analyzed by Gershuni and Zhukhovitsky [5] when the wall temperatures are constant and by Yu [6] when the wall temperatures vary linearly with the vertical distance, the flows being subjected to a transverse magnetic field.

The above studies, however, do not take into account heat transfer by radiation, which will be significant when we are concerned with space applications and higher operating temperatures. Greif, Habib and Lin [7] obtained an exact solution for the problem of fully-developed, radiating laminar convective flow in a vertical heated channel in the optically thin limit. The effect of radiation on MHD channel flow with heat transfer, however, does not seem to have received much attention. Viskanta [8] investigated the forced convection flow in a horizontal channel permeated by uniform vertical magnetic field taking radiation into account. He studied the effects of magnetic field and radiation on the temperature distribution and the rate of heat transfer in the flow but did not discuss the influence of radiation on the induced magnetic field.

The object of the present paper is to discuss the effect of radiation on the combined free and forced convection of an electrically conducting fluid flowing inside an open-ended vertical channel in the presence of a uniform transverse magnetic field. We shall, however, confine our analysis to optically thin limits.

ANALYSIS

Consider the flow of an electrically conducting fluid inside a vertical channel formed by two parallel plates (distant $2L$ apart) whose surface temperatures vary linearly along the vertical direction, taken as z-axis. We take the origin at the centre of the channel and we assume that a uniform magnetic field B_0 acts normal to the plates.

For the fully developed laminar flow in a uniform transverse magnetic field, the velocity and the induced magnetic field have only a vertical component, and all of the physical variables except temperature and pressure are functions of y, y being the horizontal co-ordinate normal to the plates. The temperature inside the fluid can be written as

$$T = T^*(y) + Nz \tag{1}$$

where N is the vertical temperature gradient. The momentum equations in the y and z directions give

$$\frac{\partial p}{\partial y} + \frac{B}{\mu}\frac{\mathrm{d}B}{\mathrm{d}y} = 0 \tag{2}$$

$$\nu\frac{\mathrm{d}^2 v}{\mathrm{d}y^2} + \frac{B_0}{\rho\mu}\frac{\mathrm{d}B}{\mathrm{d}y} + g\beta(\theta^* + Nz) - \frac{1}{\rho}\frac{\partial p}{\partial z} = 0 \tag{3}$$

the equation of continuity being identically satisfied. The energy and the magnetic induction equations reduce to

$$Nv = \alpha\frac{\mathrm{d}^2\theta^*}{\mathrm{d}y^2} - \frac{1}{\rho c_p}\frac{\partial q_R}{\partial y} \tag{4}$$

$$\frac{\mathrm{d}^2 B}{\mathrm{d}y^2} + \sigma\mu B_0\frac{\mathrm{d}v}{\mathrm{d}y} = 0 \tag{5}$$

where in (4) we have neglected the viscous and ohmic dissipation and MKS units are used in the above equations. In the optically thin limit, the fluid does not absorb its own emitted radiation. This means that there is no self-absorption but the fluid does absorb radiation emitted by the boundaries. Cogley, Vincenti and Gilles [9] showed that in the optically thin limit for a non-grey gas near equilibrium, the following relation holds

$$\frac{\partial q_R}{\partial y} = 4(T - T_w)\int_0^\infty K_{\lambda w}(\mathrm{d}e_{b\lambda}/\mathrm{d}T)_w\,\mathrm{d}\lambda \tag{6}$$

where K_λ is the absorption coefficient, $e_{b\lambda}$ is the Planck function and the subscript w refers to values at the wall. Further simplifications can be made concerning the spectral properties of radiating gases (Tien [10]) but are not necessary for our investigation. Substitution of equation (6) in (4) yields

$$Nv = \alpha\frac{\mathrm{d}^2\theta^*}{\mathrm{d}y^2} - C\theta^* \tag{7}$$

where

$$C = \frac{4}{\rho c_p}\int_0^\infty K_{\lambda 0}(\mathrm{d}e_{b\lambda}/\mathrm{d}T)_0\,\mathrm{d}\lambda. \tag{8}$$

Here the subscript 0 indicates that all quantities have been evaluated at the reference temperature T_{w0} defined in the nomenclature. Hence our study will be limited to small variations in wall temperature.

Integrating (2) with respect to y, we have

$$p = -\frac{B^2}{2\mu} + f(z). \tag{9}$$

Substitution of (9) in (3) gives

$$\nu\frac{\mathrm{d}^2 v}{\mathrm{d}y^2} + \frac{B_0}{\rho\mu}\frac{\mathrm{d}B}{\mathrm{d}y} + g\beta\theta^* = \frac{1}{\rho}\frac{\mathrm{d}f}{\mathrm{d}z} - g\beta Nz. \tag{10}$$

Since the right hand side of (10) is a function of z only and the left hand side is a function of y only, each side must be equal to a constant C_1. Thus

$$\nu\frac{\mathrm{d}^2 v}{\mathrm{d}y^2} + \frac{B_0}{\rho\mu}\frac{\mathrm{d}B}{\mathrm{d}y} + g\beta\theta^* = C_1. \tag{11}$$

The constant C_1 depends on the physics of the problem. It may be determined from either the end conditions of pressure, to which the channel is subjected, or from the mass flow through the channel.

We now introduce the following dimensionless quantities

$$\eta = \frac{y}{L},\quad u = \frac{Lv}{\alpha},\quad t = -\frac{\theta^*}{NL},\quad b = \frac{B}{B_0},$$

$$\begin{aligned} M &= \text{Hartmann number} = B_0 L(\sigma/\rho\nu)^{1/2}, \\ Ra &= \text{Rayleigh number} = g\beta NL^4/\nu\alpha, \\ Pm &= \text{magnetic Prandtl number} = \alpha\sigma\mu. \end{aligned} \tag{12}$$

Using (12), equations (11), (5) and (7) become

$$\frac{\mathrm{d}^2 u}{\mathrm{d}\eta^2} + \frac{M^2}{Pm}\frac{\mathrm{d}b}{\mathrm{d}\eta} - Ra\,.\,t = C_2 \tag{13}$$

$$\frac{1}{Pm}\frac{\mathrm{d}^2 b}{\mathrm{d}\eta^2} + \frac{\mathrm{d}u}{\mathrm{d}\eta} = 0 \tag{14}$$

$$\frac{\mathrm{d}^2 t}{\mathrm{d}\eta^2} - F\,.\,t = -u \tag{15}$$

respectively, where

$$F = \frac{L^2 C}{\alpha},\qquad C_2 = \frac{C_1 L^3}{\alpha\nu}. \tag{16}$$

The boundary conditions for velocity and temperature are

$$u = t = 0 \quad \text{at} \quad \eta = \pm 1. \tag{17}$$

Since the channel walls are assumed electrically non-conducting, the magnetic boundary conditions are

$$b = 0 \quad \text{at} \quad \eta = \pm 1. \tag{18}$$

Integration of (14) gives

$$\frac{1}{Pm}\frac{db}{d\eta}+u = \text{constant} = C_3. \tag{19}$$

Elimination of u and b from (13), (15) and (19) then yields

$$\frac{d^4t}{d\eta^4}-(F+M^2)\frac{d^2t}{d\eta^2}+(M^2F+Ra)t = C_4$$

where $C_4 = M^2C_3 - C_2$. From the above equation, the solution of $t(\eta)$ satisfying the boundary conditions (17) is easily obtained. Having found t, one can determine u and b from (15) and (19) using the boundary conditions (17) and (18). These solutions are

$$t(\eta) = \frac{C_4}{(M^2F+Ra)}\cdot\left[1-\frac{K_2^2}{(K_2^2-K_1^2)}\cdot\frac{\cosh K_1\eta}{\cosh K_1}+\frac{K_1^2}{(K_2^2-K_1^2)}\cdot\frac{\cosh K_2\eta}{\cosh K_2}\right] \tag{20}$$

$$u(\eta) = \frac{C_4}{(M^2F+Ra)}\cdot\left[F-\frac{K_2^2(F-K_1^2)}{(K_2^2-K_1^2)}\cdot\frac{\cosh K_1\eta}{\cosh K_1}+\frac{K_1^2(F-K_2^2)}{(K_2^2-K_1^2)}\cdot\frac{\cosh K_2\eta}{\cosh K_2}\right] \tag{21}$$

$$b(\eta) = \frac{C_4 Pm}{(M^2F+Ra)(K_2^2-K_1^2)}\cdot\left[\frac{K_2^2(F-K_1^2)}{K_1}\cdot\left\{\frac{\sinh K_1\eta}{\cosh K_1}-\eta\tanh K_1\right\}+\frac{K_1^2(F-K_2^2)}{K_2}\cdot\left\{\eta\tanh K_2-\frac{\sinh K_2\eta}{\cosh K_2}\right\}\right] \tag{22}$$

where

$$\begin{aligned} K_1 &= [(F+M^2)/2+\{(F-M^2)^2-4Ra\}^{1/2}/2]^{1/2} \\ K_2 &= [(F+M^2)/2-\{(F-M^2)^2-4Ra\}^{1/2}/2]^{1/2}. \end{aligned} \tag{23}$$

The non-dimensional flow rate w and the heat-transfer coefficient h (at the wall $\eta = 1$) due to thermal conduction are given by

$$\begin{aligned} w &= \int_{-1}^{1} u\,d\eta \\ &= \frac{2C_4}{(M^2F+Ra)(K_2^2-K_1^2)}\cdot\left[F(K_2^2-K_1^2)-\frac{K_2^2(F-K_1^2)}{K_1}.\tanh K_1+\frac{K_1^2(F-K_2^2)}{K_2}\tanh K_2\right] \end{aligned} \tag{24}$$

$$h = -\left(\frac{dt}{d\eta}\right)_{\eta=1} = \frac{C_4K_1^2K_2^2}{(M^2F+Ra)(K_2^2-K_1^2)}\cdot\left[\frac{\tanh K_1}{K_1}-\frac{\tanh K_2}{K_2}\right]. \tag{25}$$

increase in the induced field $b(\eta)$ with increase in F. Such a trend is reflected in Fig. 5 which shows b/Pm against η for various values of F subject to the above mentioned values of Ra, M^2 and C_4. Figure 4 shows that b/Pm decreases with increase in M^2 (with $F = 1$, $Ra = 1$, $C_4 = 1$). It may be noted that large values of Pm will correspond to large values of the induced magnetic field. Further, since the temperature inside the fluid increases with increase in M for fixed F, Ra and C_4, it follows that t decreases with increase in M as shown in Fig. 2. As for the flow rate given by (24), we may note that since u increases with F (for fixed M, Ra and C_4) w also increases with F. Further since the

RESULTS AND DISCUSSIONS

Radiation tends to increase the rate of heat transport to the gas, thereby increasing the temperature of the gas. Thus the effect of radiation is to reduce the influence of natural convection by causing a reduction in the temperature difference between the fluid and the channel walls. This is shown in Fig. 3 where t decreases with increase in F with $Ra = 1$, $M^2 = 10$, $C_4 = 1$. Again due to this effect, the reduction in velocity occurring in a heated upflow is less for a radiating fluid than that for a non-radiating fluid. This explains why the velocity $u(\eta)$ at a point increases with increase in F for fixed values of Ra, M^2 and C_4 as shown in Fig. 1, where we have taken $Ra = 1$, $M^2 = 10$ and $C_4 = 1$. This increase in velocity in its turn tends to pull the magnetic lines of force thereby causing an velocity profile becomes progressively flatter with increase in the magnetic parameter M (for fixed F, Ra and C_4), w decreases with increase in M. These results are shown in Table 1 for $Ra = 1$ and $C_4 = 1$. The corresponding results for h, the rate of heat transfer at the wall given by (25) are shown in Table 2 for $Ra = 1$ and $C_4 = 1$.

We conclude our discussion with an interesting observation on the effect of radiation on the flow in the unstable situation $N < 0$. In this case there is heating from below so that $Ra < 0$ as can be seen from (12). Although the solutions for $t(\eta)$, $u(\eta)$, and $b(\eta)$ given by (20)–(22) are finite for all values of F, Ra and M^2 with $Ra > 0$, these functions may become infinite for $Ra < 0$ such that K_2 becomes a pure imaginary number given by $K_2' = iK_2$, K_2' being

Table 1. Variation of flow rate with radiation and magnetic field

M^2	F 1	2	3	4	5	6
10	0·134033	0·134679	0·135093	0·135380	0·135592	0·135754
20	0·076726	0·076929	0·077058	0·077148	0·077214	0·077264
30	0·054053	0·054151	0·054213	0·054256	0·054238	0·054312

Table 2. Variation of heat-transfer rate at the wall with F and M

M^2	F 1	2	3	4	5	6
10	0·048561	0·038499	0·032031	0·027513	0·024172	0·021597
20	0·027991	0·022281	0·018617	0·016058	0·014165	0·012705
30	0·019809	0·015819	0·013259	0·011471	0·010147	0·009124

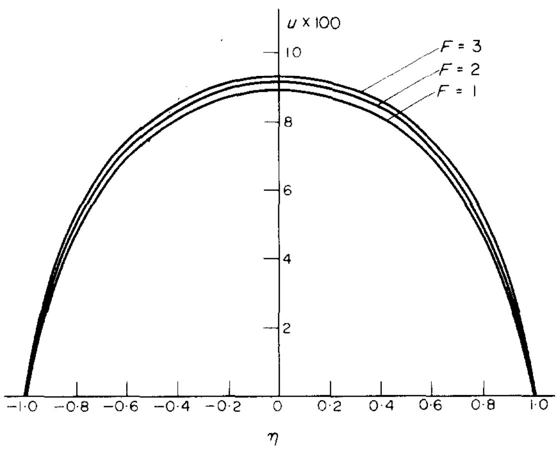

FIG. 1. Effect of radiation on velocity for $M^2 = 10$.

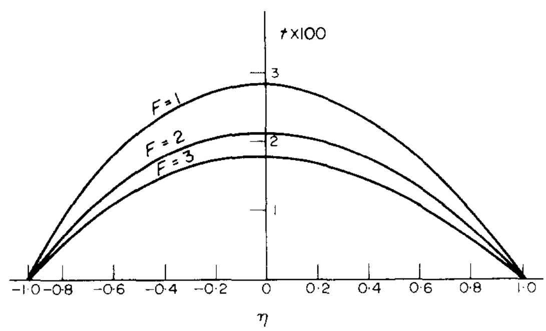

FIG. 3. Effect of different radiation parameters on temperature for $M^2 = 10$.

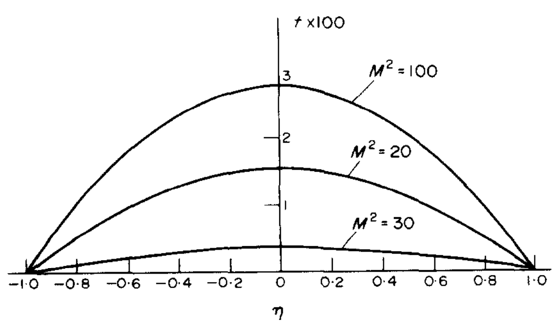

FIG. 2. Temperature distribution for different values of Hartmann number for $F = 1$.

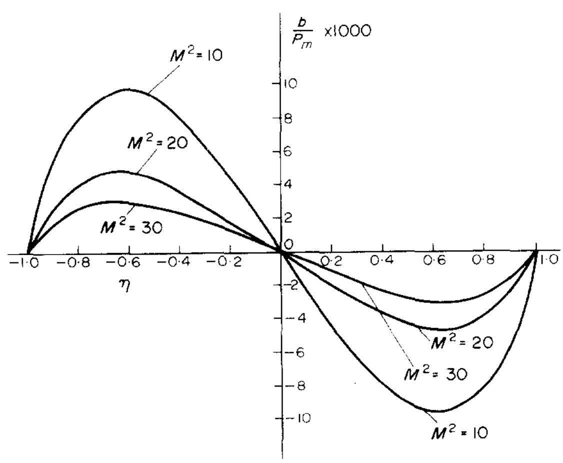

FIG. 4. Effect of Hartmann number on induced field for $F = 1$.

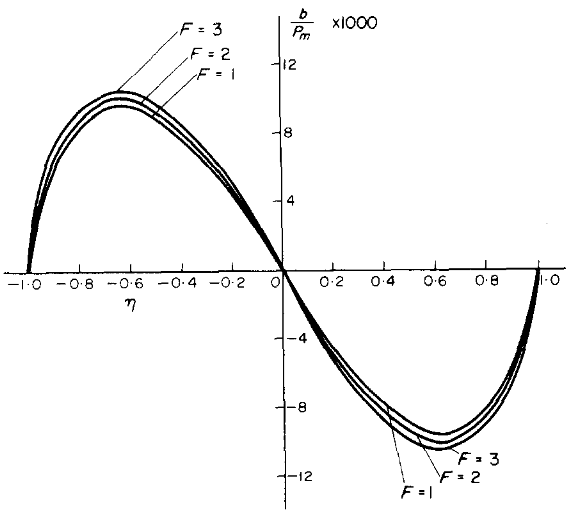

FIG. 5. Effect of radiation on induced field for $M^2 = 10$.

real. Examination of the solutions (20)–(22) reveals that there are critical values of K_2' (viz. $K_2' = n\pi/2$, n being an odd integer), such that $\cosh K_2 = \cos K_2'$ vanishes and $t(\eta)$, $u(\eta)$ and $b(\eta)$ become infinite. This is a result of the emergence of instability. Equating iK_2 to $\pi/2$ (corresponding to the lowest eigenvalue $n = 1$) with K_2 given by (23), the critical Rayleigh number at the onset of instability is given by

$$-Ra_c = \frac{\pi^2}{4}\left[\frac{\pi^2}{4} + F + M^2\right] + FM^2 \tag{26}$$

with $Ra_c < 0$. This shows that the critical Rayleigh number not only increases with increase in the Hartmann number M but also increases with increase in the radiation parameter F. Thus radiation exerts a stabilizing influence on the flow which is consistent with the fact that radiation tends to reduce the effect of natural convection.

Finally we remark that although our analysis is confined to optically thin limit flows, such a study will be useful in increased understanding of the flow phenomenon which can then be used to study more complex phenomena involving radiation.

Acknowledgements—We sincerely thank Mr. S. Chatterjee for his help in computer programming.

REFERENCES

1. M. F. Romig, The influence of electric and magnetic fields on heat transfer to electrically conducting fluids, *Advances in Heat Transfer*, edited by T. F. Irvine, Jr. and J. P. Hartnett, Vol. 1. Academic Press, New York (1964).
2. R. Siegel, Effect of magnetic field on forced convective heat transfer in a parallel plate channel, *J. Appl. Mech.* **25**, 415 (1958).
3. M. Perlmutter and R. Siegel, Heat transfer to an electrically conducting fluid flowing in a channel with a transverse magnetic field, NASA TN-D-875 (August 1961).
4. R. A. Alpher, Heat transfer in magnetohydrodynamic flow between parallel plates, *Int. J. Heat Mass Transfer* **3**, 108 (1961).
5. G. Z. Gershuni and E. M. Zhukhovitskii, Stationary convection flow of an electrically conducting liquid between parallel plates in a magnetic field, *Sov. Phys. JETP* **34**(7), 461 (1958).
6. C. P. Yu, Combined forced and free convection channel flows in magnetohydrodynamics, *AIAA Jl* **3**, 1184 (1965).
7. R. Greif, I. S. Habib and J. C. Lin, Laminar convection of a radiating gas in a vertical channel, *J. Fluid Mech.* **46**, 513 (1971).
8. R. Viskanta, Effect of transverse magnetic field on heat transfer to an electrically conducting and thermally radiating fluid flowing in a parallel plate channel, *Z. Angew. Math. Phys.* **14**, 353 (1963).
9. A. C. Cogley, W. C. Vincenti and S. E. Gilles, Differential approximation for radiative transfer in a non-grey gas near equilibrium, *AIAA Jl* **6**, 551 (1968).
10. C. L. Tien, Thermal radiation properties of gases, *Advances in Heat Transfer*, Vol. 5. Academic Press, New York (1968).

EFFET DU RAYONNEMENT SUR L'A CONVECTION HYDROMAGNETIQUE DANS UNE CONDUITE VERTICALE

Résumé—On considère l'effet du rayonnement sur la convection mixte d'un fluide électriquement conducteur dans un canal vertical ouvert aux extrémités et soumis à un champ magnétique uniforme et transversal. On obtient les solutions analytiques pour la vitesse, la température et pour le champ magnétique induit, ceci dans le cas d'une épaisseur optique mince et d'une température de paroi variant linéairement avec la distance verticale. On trouve que le rayonnement tend à accroitre le flux de transport thermique et de réduire en conséquence l'effet de la convection naturelle. La vitesse et le champ magnétique induit augmentent et la différence de température entre la paroi et le fluide diminue lorsque le paramètre de rayonnement croît. Dans la situation instable qui correspond au chauffage du canal à partir du bas, le rayonnement et le champ magnétique exercent tous deux une influence stabilisante sur l'écoulement.

STRAHLUNGSEINFLUSS BEI HYDROMAGNETISCHER KONVEKTION IN EINEM VERTIKALEN KANAL

Zusammenfassung—Es wird der Strahlungseinfluß auf den gekoppelten freien und erzwungenen Konvektionsstrom eines elektrisch leitenden Fluides in einem vertikalen Kanal mit offenem Ende betrachtet, wobei das Fluid einem gleichförmigen transversalen Magnetfeld ausgesetzt ist. Eine geschlossene Form von Lösungen erhält man für die Geschwindigkeit, die Temperatur und das induzierte Magnetfeld im optisch dünnen Grenzbereich, für den sich die Wandtemperatur linear mit der vertikalen Entfernung verändert. Es zeigt sich, daß die Strahlung dazu führt, den Anteil des Wärmetransportes zu erhöhen und dabei gleichzeitig den Anteil der natürlichen Konvektion zu verringern. Geschwindigkeit und induziertes Magnetfeld nehmen zu, die Temperaturdifferenz zwischen Wand und Fluid nimmt mit der Größe des Strahlungs parameters ab. Im instabilen Bereich der Beheizung des Kanals von unten üben die Strahlung und das Magnetfeld auf die Strömung einen stabilisierenden Einfluß aus.

ВЛИЯНИЕ ИЗЛУЧЕНИЯ НА КОНВЕКЦИЮ В ВЕРТИКАЛЬНОМ КАНАЛЕ ПРИ НАЛОЖЕНИИ МАГНИТНОГО ПОЛЯ

Аннотация — Рассматривается влияние излучения на совместную свободную и вынужденную конвекцию в потоке электропроводной жидкости в открытом вертикальном канале при наложении однородного поперечного магнитного поля. Получены замкнутые решения для скорости, температуры и наведенного магнитного поля в случае оптически тонкой среды при линейном изменении температуры стенок по вертикали. Установлено, что излучение интенсифицирует перенос тепла к жидкости, уменьшая таким образом влияние естественной конвекции. Скорость течения и величина магнитного поля возрастают, а разность температур стенки и жидкости уменьшается с увеличением параметра излучения. В нестационарных условиях, соответствующих нагреву канала снизу, излучение и магнитное поле оказывают стабилизирующее влияние на поток жидкости.